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After the Kaldor’s Stylized Facts:
A Theory of the Labor Share’s Fall

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1. Introduction

Over last several years, we have witnessed the increasing number of research that indicate the decline of the labor share. This phenomenon has been clearly confirmed by Autor et al. (2017). One of their important findings is that “a fall in the labor share will be driven largely by between-firm reallocation rather than a fall in the unweighted mean labor share within firms.” In order to explain this fact, they have presented the superstar-firm theory; since the most productive firms with a low share of labor become increasingly dominate industry, the aggregated labor share will tend to fall. Although their theory clearly has many aspects of dynamics, it is totally analyzed in the frame of statics. We definitely need a dynamic model for a theoretical explanation of the phenomenon.

However, we cannot apply the macro-dynamic models used in many macro growth research, because they are mainly built for focusing on the “Kaldor facts” and firmly based on the Solow growth model. Since the core of the Kaldor facts is now empirically refuted, we definitely need an alternative dynamic model to explain the new phenomenon.

I would like to propose here a multisector optimal growth model as the model for giving a firm theoretical base of the superstar-firm theory. The multisector optimal growth models were intensively studied in 1970s and 1980s by L. McKenzie, among others, under the title of Turnpike Theory. Furthermore, based on Turnpike Theory, Takahashi (1985) has established the existence and global stability of the optimal stationary path in the multisector optimal growth model with one-pure consumption good and n goods sectors. I will briefly explain how this approach could be applicable to the superstar-firm theory below.

2. The Model and Assumptions

We start from the following neoclassical multisector optimal growth model with a sector specific TFP term (in other words, the sector specific Hicks neutral technical progress). The
index “i” stands for the ith sector. Instead of calling it the ith “sector”, we may call it the ith “industry” or the ith “firm.” It is also noteworthy that the number of sector is finite but can take a large number, say a million. We assume that one pure-consumption-good producing sector and n good-producing sectors, which will produce goods used for both consumption and capital goods. Our model will be presented as follows:

\[
\text{Maximize } \sum_{i=0}^{\infty} \rho^i u (c_0(t), c_1(t), c_2(t), \ldots, c_n(t)) = \sum_{i=0}^{\infty} \rho^i u (c_0(t), c(t))(0 < \rho < 1)
\]

subject to \( k(0) = \bar{k} \)

\[
y_i(t) - c_1(t) + k_1(t) - \delta_i k_i(t) - (1 + n) k_i(t) = 0 \quad (i = 1, 2, \ldots, n)
\]

\[
c_0(t) = A_0(t) f^*(k_0(t), k_2(t), \ldots, k_0(t), \ell_0(t))
\]

\[
y_i(t) = A_i(t) f^*(k_0(t), k_2(t), \ldots, k_0(t), \ell_i(t)) \quad (i = 1, 2, \ldots, n)
\]

\[
\sum_{j=0}^{n} \ell_j(t) = 1 \quad (t = 0, 1, 2, \ldots)
\]

\[
\sum_{j=0}^{n} k_j(t) = k(t) \quad (i = 1, \ldots, n; t = 0, 1, 2, \ldots).
\]

The symbols have the following meanings:

- \( n \) = population growth rate \((0 \leq n \leq 1)\),
- \( \gamma \) = subjective discount factor,
- \( \rho \) = \((1 + n)/(1 + \gamma)\) \((n < \gamma)\),
- \( u \) = representative individual utility function,
- \( c_i(t) \in \mathbb{R}, (i = 0, 1, \ldots, n) \) = per-capita consumption goods consumed in period \( t \),
- \( y_i(t) \in \mathbb{R}, (i = 1, \ldots, n) \) = capital good per-capita production vector in period \( t \),
- \( k(0) \in \mathbb{R}_+ \) = initial capital stock vector,
- \( f^*: \mathbb{R}_+^n \rightarrow \mathbb{R}_+ \) = per-capita production function of the j sector,
- \( k_j(t) \) = capital good i used in sector j in period \( t \),
- \( \ell_j(t) \) = labor input used in sector j in period \( t \),
- \( \delta_i \) = depreciation rate of capital good i \((0 < \delta_i < 1)\).

We make following assumptions for avoiding further complications:

**Assumption 1.**

1) The utility function \( u(\cdot) \) is defined on \( \mathbb{R}_+ \) so that
\[ u(c_a(t), c(t)) = \left( \prod_{i=0}^{n} c_i(t) \right)^{\tau} \text{ where } \tau \in (-\infty, 1) \text{ and } \tau \neq 0. \]

2) \( A_i(t) = (1 + \alpha_t) A_i(0) \) where \( \alpha_t \) is a rate of the Hicks neutral technical progress (or the rate of TFP growth) of the \( i \) th sector and given as \( |\alpha| < 1 \).

**Assumption 2.**

1) All the goods are produced non-jointly with the per-capita production functions \( f^i(\cdot) \) \( (i = 0, 1, \ldots, n) \) which are defined on \( \mathbb{R}^{n+1} \), homogeneous of degree one, strictly quasi-concave and continuously differentiable for a positive inputs.

2) Any goods \( j \) \( (j = 0, 1, \ldots, n) \) cannot be produced unless \( k_j > 0 \) for some \( i = 1, \ldots, n \).

3) Labor must be used directly in each sector. If labor input of some sector is zero, its sector’s output is zero.

Now dividing each output variable by its own rate of technical progress \( A_i(t) \) yields the followings:

\[ \tilde{y}_i(t) = \frac{y_i(t)}{A_i(t)} \quad (i = 1, \ldots, n), \quad \tilde{c}_0(t) = \frac{c_0(t)}{A_0(t)}. \]

The objective function is also rewritten as follows:

\[ \sum_{j=0}^{n} \left( 1 + \gamma \right)^{\tau} \left( \prod_{i=0}^{n} c_i(t) \right)^{\tau} = \sum_{j=0}^{n} \left( 1 + \gamma \right)^{\tau} \left( \prod_{i=0}^{n} c_i(t) \right)^{\tau} \]

**Assumption 3.** \( \prod_{i=0}^{n} A_i(0) L(0) = 1 \) and \( 0 < \eta < 1 \).

Now under Assumption 3 the original model can be rewritten as the following per-capita efficiency-unit optimal problem:

**The Per-capita Efficiency-unit Optimal Problem:**

\[
\text{Max} \sum_{j=0}^{n} \eta u(\tilde{c}_0(t), \tilde{c}(t)) \quad \text{where } \eta = \frac{(1 + g)^{\tau} \prod_{i=0}^{n} (1 + \alpha_i)^{\tau}}{(1 + \gamma)^{\tau}},
\]

\[
s.t \quad \tilde{k}_i(0) = \tilde{k}_i(i = 1, \ldots, n),
\]

\[ c_0(t) = f^\ast(k_{10}(t), k_{20}(t), \ldots, k_{n0}, c_0(t)) \]
\[
\hat{y}_i(t) = f^*(k_{i1}(t), k_{i2}(t), \ldots, k_{in}(t)) \quad (i = 1, \ldots, n)
\]

(8)

\[
\hat{y}(t) - \hat{c}(t) + (1 - \Delta) \mathbf{k}(t) - (1 + g) \mathbf{k}(t + 1) = 0,
\]

(9)

where

\[
\hat{y}(t) = (\hat{y}_1(t), \hat{y}_2(t), \ldots, \hat{y}_n(t)), \mathbf{k}(t) = (k_1(t), k_2(t), \ldots, k_n(t)), \text{and } \Delta = \begin{pmatrix}
\delta_1 & 0 \\
0 & \delta_n
\end{pmatrix}.
\]

\[
\sum_{i=0}^{n} \ell_i(t) = 1 \quad \text{and} \quad \sum_{j=0}^{n} k_{ji}(t) = k_i(t) \quad (i = 1, \ldots, n).
\]

(10)

In order to apply Turnpike Theory developed by McKenzie and Scheinkman, among others to the above optimization model, first of all, we need to transform the model into the following reduced-form optimal growth model expressed by initial and terminal capital stock vector denoted by \((\mathbf{k}(t), \mathbf{k}(t + 1))\).

**The Reduced-form Optimal Growth Model:**

\[
\begin{cases}
\max_{\mathbf{k}(0)} \sum_{t=0}^{\infty} \rho^t V(\mathbf{k}(t), \mathbf{k}(t + 1)) & (0 < \rho < 1) \\
\text{s.t.} & \\
(\mathbf{k}(t), \mathbf{k}(t + 1)) \in \mathbf{D} \text{ and } \mathbf{k}(0) = \mathbf{k^0}
\end{cases}
\]

where \(V: \mathbf{D} \subset \mathbb{R}^n_{+} \rightarrow \mathbb{R}_{+} \), \(\mathbf{k}(t) \in \mathbb{R}^n_{+} \) and \(\mathbf{k}(0)\) is a given initial stock vector.

Under several assumptions concerned with the model, the following existence and global stability theorems were proved by McKenzie (1983, 1984):

1) **The existence theorem:** There is a unique stationary optimal path \(\{\mathbf{k}^\rho\}^\infty\), supported by price vectors \(\mathbf{p}(t) = \rho^t \mathbf{q}\) in the sense that

\[
V(\mathbf{k}, \mathbf{k^0}) + \rho \mathbf{qk^0} - \mathbf{kq}^t \geq V(\mathbf{z}, \mathbf{w}) + \rho \mathbf{qw} - \mathbf{qz} \text{ for all } (\mathbf{z}, \mathbf{w}) \in \mathbf{D}.
\]

2) **The global stability theorem:** There is a \(\rho' > 0\) such that for \(\rho \in [\rho', 1)\), any optimal path \(\{\mathbf{k}'(t)\}^\infty\) with the sufficient initial stocks asymptotically converges to the optimal stationary path \(\{\mathbf{k}^\rho\}^\infty\).

**3. Implications**

To apply the theorems 1) and 2) to the normalized model, we need quite a long argument as discussed in Takahashi (1985). Let’s assume that it had already been done here. Then we have
the following interesting implications:

i) From the existence theorem, we can show that the unique optimal stationary output vector \((\tilde{c}_i, \tilde{y}_1, ..., \tilde{y}_n)\) exists. Since \(\tilde{y}_i = \frac{y_i(t)}{A_i(t)} (i = 0, 1, ..., n)\), where if \(i = 0\), \(\tilde{y}_0 = \tilde{c}_0\), it follows that \(y_i(t) = y_i^* A_i(t) = (1 + \alpha_i) A_i(0) \tilde{y}_0^*\). Hence the original optimal series of the output \(y_i(t)\) is growing at the rate of its own sector’s technical progress; \(\alpha_i\). It is noteworthy that each sector will grow at its own rate of sector’s TFP growth along optimal steady states; it exhibits unbalanced growth among sectors. So the sectors with the higher TFP growth rate will gradually dominate other sectors on the optimal steady state path.

ii) Due to the global stability theorem, any sector’s optimal path which starts from arbitrarily given initial stock should asymptotically converge to its own optimal steady state, which will grow at the sector’s specific TFP growth rate.

4. An Expected Result

Now let us suppose that each sector’s production technology is the Cobb-Douglas function. In other words, each sector’s labor share is constant but different across sectors. Furthermore, suppose that the sector (or firm) with the higher TFP growth rate will have the lower labor share. This property is theoretically and empirically established by Autor et al. (2017). It will be also confirmed for the US industrial sectors by the following additional figure below. And as a result, through the dynamic process of domination by the sectors (or firms) with the higher TFP rate, the aggregated labor share should gradually decline.

References


References


Average TFP Growth Rate and Labor Share from 1999 to 2010 in the US Industries: Corr. = - 0.39