Exchange Rate Changes and the Trade Balance : Derivation of Generalized Marshall-Lerner Condition

【Research Note】

Exchange Rate Changes and the Trade Balance : Derivation of Generalized Marshall-Lerner Condition *

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【Abstract】

In international economics, it has been well-known that when a nation's currency appreciates (depreciates) the international trade balance decreases (increases), provided that import and export price-elasticities satisfy the so-called Marshall-Lerner (M-L) condition. This paper analyses the case in a more general setting and derives a "generalized M-L condition", and shows that it includes the conventional M-L condition as a special case, as shown in Table 1 (on page 8, which appeared originally in Okabe 1986). The analysis also reveals that home country's (Japanese) trade balance needs to be expressed not in foreign currency (dollar) but in home currency (yen) to effectively capture the external adjustment process. After this requirement was officially recognized, the authorities began in 1987 to publish the statistics of Japanese international trade and finance in both dollar and yen, and after 1996 only in yen.

Key words: Exchange rate, trade balance, home currency, foreign currency, generalized Marshall-Lerner Condition

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* This is a very brief summary of the main points of my earlier paper in Japanese (Okabe 2011), which was presented at the biannual conference of the Japanese Economic Association in May 2011. For detailed literature review, related arguments and empirical ecidence, as well as full bibliography, please refer to the original paper. I am greateful to the discussant Professor Ryuhei Wakasugi (Kyoto University) for valuable comments on the paper. I also thank the refrees of the *International & Regional Studies* for various suggestions.

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Introduction

This is a brief note to show the essence of my earlier paper (Okabe 2011), whose core arguments and the rigorous proof of the main arguments are, to my knowledge, not found anywhere in the literature of international economics in English[1].

The theme here is the so-called Marshall-Lerner condition (M-L conditon, hereafter) pertaining to exchange rate changes and the international trade. The M-L condition states that, if the exchange rate change should bring about an expected change to the international trade balance (e.g. the appreciation of a currency to reduce the trade balance), the sum of export price-elasticity and import price-elasticity (in absolute value) needs to be greater than unity (one).

In Section 1, the significance of M-L condition in international economics is briefly reviewed. Section 2 points out four neglected issues when we deal with the M-L condition. In Section 3, we derive generalized M-L condition and show that the conventional M-L condition is a special case of "generalized M-L condition". Section 4 points out some related issues and implications.

1. Significance of the Marshall-Lerner Condition in International Economics

In international economics, there are a variety of analyses containing not only import and export prices, exchange rates, and trade balances, but also terms of trade, foreign currency market, and income level, for instance. But surprizingly, in many cases the conclusion of the analyses reduces to the so-called M-L condition, although that is called by different names (Okabe 2011: 20-21), such as Bickerdike-Robinson-Metzler condition, Robinson-Metzler-Bickerdike equation, Marshall-Lerner-Robinson condition (Kenen 1985: 325), or the stability condition of international trade (Bhagwati et al. 1998: 141). This implies that the M-L condition may be said to be one of the conerstones in the entire international economics theory.

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[1] If I utilize Google Scholar to retrieve academic papers on the internet relating to "generalized Marshall-Lerner condition", I have found that there are about 1,680 items (as of January 2020). But none of them seems to treat the generalized M-L condition in the same way as I describe below.

2. Four Neglected Aspects

The elegance of the M-L condition naturally owes to some simple assumptions. Kenen (1985: 327), for instance, points out that the assumptions are (1) international trade is balanced initially, (2) home and foreign prices are constant, and (3) incomes are constant regardless of exchange rate changes. These are all valid assumptions, especially when we deal with a simple (partial equilibrium) theory.

However, when we are concerned with actual economy and economic policy, we seem to have often applied the simple M-L condition and have generally failed to take the following four important aspects into consideration. Namely there has been: (1) lack of understanding of initial position of trade balance (surplus or deficit), (2) lack of understanding of the selection of the currency to express the trade balance, (3) failure to simultaneously derive conditions for both short-term and long-term, and (4) failure to integrate the export price-raising behavior when home currency appreciates.[2] In regard to the problem (1), Krugman[3] et al. (2018: 532) clearly recognize the limitation of the condition and point out "if the current account is not zero initially, the [Marshall-Lerner] condition becomes more complex." But it does not seem there exist some generalized condition in the literature (see footnote 1). The derivation of a general condition is certainly complicated, but turns out surprisingly simple, and not complex as Krugman et al implied. The result is shown below.

In order to solve the above four problems simultaneously, Okabe (2011) introduced a model as is explained in the next section. In this note, however, we take up only issues (1) and (2) , and show the solutions. For solving issues (3)[4] and (4) , the same model can be used satisfactorily as well (for the detail, see Okabe 2011: 24-25).

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[2] When the home currency (yen) appreciates, Japanese exporters have traditionally tended to raise export (dollar) prices by about 60~70% in order to cover the reduction of export income in yen. This was possible since Japanese export products have enjoyed some "quality" elements in international markets. While Japan has usually been assumed to be a price-taker for her imports (in dollar).

[3] The Nobel laureate in economic sciences in 2008.

[4] The short-term effect of the issue (3) is usually called the "J-curve effect" (Krugman et al. 2018: 515-516) . It means that, in the short-run, exchange rate changes bring about an inverse effect to trade balance. This phenomenon has been confirmed widely in various empirical studies.

3. Derivation of Generalized Marshall-Lerner Condition

In order to rectify the above two problems, (1) and (2), we consider a general case in two aspects: (a) we suppose a general situation where initial trade balance may or may not be balanced; (b) we consider two cases to express the trade balance: one is in home currency (yen, hereafter), and the other in foreign currency (dollar, hereafter) . These are the basic assumptions, and the resulting analysis, as shown below, was initially documented in an unpublished internal paper (Okabe 1986) when the author was a senior economist of the Bank of Japan, and only the results of the paper as summarized in Table 1, shown below on page 8, were made public in Okabe (1988) . But the full analysis was made public only recently as Okabe (2011).

Let us introduce the following variables.

B	Japan's trade balance (yen)
C	Japan's trade balance (dollar)
p	Price of Japanese product (yen)
p^*	Price of foreign product (dollar) (fixed)
M	Japan's import volume
X	Japan's export volume
π	Yen exchange rate (yen/dollar)
α	Price (dollar-base) -elasticity of foreigner's demand to Japanese product
β	Price (yen-base) -elasticity of Japan's import demand to foreign product
ε	Sensitivity of the price of Japan's exporting product to yen exchange rate fluctuation. ($1-\varepsilon$ denotes the ratio of export price increase to yen appreciation. $0 \leqq \varepsilon \leqq 1$)

(1) Yen appreciation and trade balance (in yen)

The trade balance of Japan *in yen* can be expressed by the following expression :

$$B = p \cdot X(\frac{p}{\pi}) - \pi p^* \cdot M(\pi p^*) \qquad \cdots\cdots (1)$$

The change in trade balance, when the exchange rate changes, will be given by differentiating the trade balance, B, with respect to exchange rate, π. Although the calculating process is fairly complicated and tricky, the result can be expressed as follows :

$$\frac{dB}{d\pi} = p \cdot \frac{dX}{d(p/\pi)} \cdot (\frac{1}{\pi}\frac{dp}{d\pi} - \frac{p}{\pi^2}) + \frac{dp}{d\pi} \cdot X - p^* \{\pi p^* \cdot \frac{dM}{d(\pi p^*)} + M\}$$

$$= (p/\pi) \cdot X \cdot (1 - \underbrace{\frac{dp}{p}/\frac{d\pi}{\pi}}_{\varepsilon}) \cdot \{- \underbrace{\frac{dX}{X}/\frac{d(p/\pi)}{p/\pi}}_{\alpha}\}$$

$$+ (p/\pi) \cdot X \cdot (\underbrace{\frac{dp}{p}/\frac{d\pi}{\pi}}_{\varepsilon}) + p^* M \{- \underbrace{\frac{dM}{M}/\frac{d(\pi p^*)}{\pi p^*}}_{\beta} - 1\}$$

$$= (p/\pi) \cdot X \cdot \{(1-\varepsilon)\alpha + \varepsilon - \frac{p^* M}{(p/\pi)X}(1-\beta)\} \qquad \cdots (2)$$

Resulting effects of yen appreciation (long-run effect)

The condition for yen appreciation to bring about a "normal" result on the trade balance (i.e. the reduction of the trade balance) is $dB/d\pi > 0$. That is, for the value of (2) above, the following inequality (3) needs to hold :

$$(1-\varepsilon)\alpha + \varepsilon - \frac{p^* M}{(p/\pi)X}(1-\beta) > 0 \qquad \cdots\cdots (3)$$

Here, let us consider two cases. Firstly, the case where international trade is *initially balanced*, so that $p^* M = (p/\pi)X$. In this case, the inequality (3) reduces to the following :

$$\alpha + \beta + \varepsilon(1-\alpha) > 1 \qquad \cdots\cdots\cdots \quad (4a)$$

If we assume further that Japan's export price (in yen) remains unchanged, $\varepsilon = 0$, this inequality reduces to :

$$\alpha + \beta > 1 \qquad \text{(M-L condition)} \quad \cdots \quad (4b)$$

Secondly, the case where international trade balance is initially *surplus*, so that $p^*M < (p/\pi)X$. Here, if we let m be the relative size of export to import, $(p/\pi)X / p^*M = m, \quad m > 1$, the inequality (3) reduces to the following :

$$m\{\alpha + \varepsilon(1-\alpha)\} + \beta > 1 \qquad \cdots\cdots\cdots \quad (5a)$$

If we further assume $\varepsilon = 0$, the inequality (3) is much simplified to :

$$m\alpha + \beta > 1 \qquad \cdots\cdots\cdots \quad (5b)$$

(2) Yen appreciation and trade balance (in dollar)

The trade balance of Japan *in dollar* is given by the following :

$$C = \frac{1}{\pi} \cdot B \qquad \cdots\cdots\cdots\cdots\cdots \quad (1)'$$

In this case, the change in trade balance when the exchange rate changes will be given by differentiating the trade balance, C, with respect to exchange rate, π. The result is :

$$\frac{dC}{d\pi} = \frac{1}{\pi} \cdot \frac{dB}{d\pi} - \frac{1}{\pi^2} B$$

$$= \frac{(p/\pi)X}{\pi}\{(1-\varepsilon)\alpha + \varepsilon + \frac{p^*M}{(p/\pi)X}\beta - 1\} \cdots (2)'$$

Resulting effects of yen appreciation (long-run effect)

The condition for yen appreciation to bring about "normal" result on the trade balance (i.e. the reduction of trade balance) is for the value of (2)' to be positive,

$dC/d\pi > 0$:

$$(1-\varepsilon)\alpha + \varepsilon + \frac{p^*M}{(p/\pi)X}\beta - 1 > 0 \quad \cdots (3)'$$

Here, let us take up two cases. Firstly, the case where international trade was *initially balanced*, so that $p^*M = (p/\pi)X$. In this case, the inequality (3)' reduces to the following :

$$\alpha + \beta + \varepsilon(1-\alpha) > 1 \quad \cdots\cdots\cdots (4a)$$

This inequality is the same as the earlier case where trade balance is expressed in yen. Here, if we assume further, as we did in the yen-based case, that Japan's export price (in yen) remains unchanged, we can let $\varepsilon = 0$. In this case, the inequality reduces to :

$$\alpha + \beta > 1 \qquad \text{(M-L condition)} \quad \cdots (4b)$$

This result is identical to the yen-base case, as we obtained as inequality (4b) earlier.

Second case is the one where trade balance is initially *surplus*. Here, we write again $(p/\pi)X/p^*M = m \quad (m > 1)$, the inequality (3)' can be written as :

$$\alpha + (1/m)\beta + \varepsilon(1-\alpha) > 1 \quad \cdots\cdots (5a)'$$

And if we further assume $\varepsilon = 0$, this inequality reduces to :

$$\alpha + (1/m)\beta > 1 \quad \cdots\cdots\cdots (5b)'$$

(3) Currency Appreciation and Trade Balance: Yen term vs. dollar term

So far, we have analytically drawn the effects of home currency appreciation on trade balances, and contrasted the two cases: when the trade balance is expressed in yen and in dollar. The results can be summarized as in Table 1 (assuming $\varepsilon = 0$) .

Table 1

The Marshal-Lerner Condition: Conventional and Generalized Forms

	Trade balance in yen	Trade balance in dollar
When initial trade balance is balanced (zero)	$\alpha + \beta > 1$	$\alpha + \beta > 1$
When initial trade balance is deficit or surplus	$m\alpha + \beta > 1$	$\alpha + (1/m)\beta > 1$

(Note) 1. m = export ÷ import.

2. $m > 1$: initial trade balance surplus.
   $m < 1$: initial trade balance deficit.

We can clearly see two interesting cases. First, when trade is initially balanced ($m = 1$), there is only one M-L condition, as shown in the first line of the table, regardless of the selection of currency (yen or dollar) to express international trade and trade balance.

Second, in a general situation ($m \neq 1$), as shown in the second line of the table, the required condition of the effect of currency appreciation or depreciation upon the trade balance defers. Now the required condition is expressed not by an inequality but by a pair of inequalities. And the condition varies and depends on whether trade and trade balance is expressed in yen (in home currency) or in dollar (in foreign currency). The point is that the condition now includes a new variable m, and a pair of conditions is expressed in a beautiful symmetric form.

Interestingly, we can easily confirm that, when $m = 1$ (when trade is initially balanced) the two different conditions reduce to one condition, to the conventional M-L condition (as expressed in the first line of this table). This means that the conventional M-L condition is a special case applicable only to the case where international trade is initially balanced ($m = 1$), as Kenen (1985: 327) emphasized. So the condition we have derived, shown in the second line of this table, may be called "generalized Marshall-Lerner condition" (or possibly Marshall-Lerner-Okabe

condition). The derivation of this has been in fact the core argument of the author's earlier paper in Japanese (Okabe 2011).

4. Related Arguments and Implications

In the original Japanese paper (Okabe 2011), some other arguments are also taken up and developed. They include the following: (1) we can provide a theoretical proof of the short-term reverse phenomenon (the J-curve effect) of trade balance changes, by using the same model; (2) we can provide various empirical justification of the speed and the degree of Japan's trade balance adjustment by utilizing the generalized Marshall-Lerner condition; (3) the necessity for Japan's international balance of payments to be expressed in Japanese yen, not in US dollar, in order to accurately understand the trade balance adjustment process.

Reflecting the above (3), the Japanese authorities (Bank of Japan and the Ministry of Finance) actually changed how international financial statistics are compiled and published. Up to the year 1986, all the international trade and financial statistics had been published in US dollar terms. But after recognizing the above (3), the authorities began in 1987 to publish the statistics in both US dollar and Japanese yen terms. And after 1996, they consequently suspended the use of dollar-based statistics and only the yen base statistics has been published ever since.

【Referance】

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