『経済研究』（明治学院大学）第 154 号　2017 年

Sector-Specific Logistic Technological Progress in a Two-Sector Optimal Growth Model*

Kazunobu Muro†‡§

Abstract

To verify "Baumol's cost disease" and the Kuznets fact, we construct a two-sector optimal growth model with service and manufacturing sector-specific logistic technological progress as well as a logistic population growth. We show three main results. First, the logistic population growth does not affect the steady state. On the other hand, the logistic sector-specific technology of service and of manufacturing goods industry has an asymmetric effect on the steady state. Second, the carry capacity of the service sector-specific technology has a negative effect on the relative price of service at the steady state while it does not affect the capital per capita at the steady state. On the other hand, the carry capacity of the manufacturing sector-specific technology has a positive effect on the relative price of service and capital stock per capita at the steady state. Finally, the dynamical system has saddle point stability.

Keyword: *Baumol's cost disease; GDP function; Logistic Function; Carry Capacity*
JEL classification: E13, O11, O41

1 Introduction

Although a new technology offers an increase in efficiency, it does not spread instantly, but is adopted at changing rate. These adoption processes are modeled by a logistic curve. First, the rate of adoption is slow, as a new technology must struggle to replace a mature one. The rate of adoption increases, usually exponentially until physical or other limits slow the adoption. Adoption is a kind of social epidemic. Technological innovations do not usually distribute themselves evenly through time, but cluster in spurts or innovation waves. We emphasize that the rate of technological progress in the service and manufacturing industry is different. Furthermore, the upper limit of technology in the service and manufacturing industry is also different. The purpose of this paper is to verify "Baumol's cost disease" and the Kuznets fact by considering the sector-specific logistic technological progress. Baumol (1967) [3] studies a progressive sector which uses new technology and a stagnant sector which uses labor as the only input. He insists that the production costs and prices of the stagnant sector rises indefinitely, a process known as Baumol's cost disease, and labor moves in the direction of the stagnant sector. Now, you can reinterpret the progressive sector as manufacturing industry and the stagnant sector as service industry. Although productivities in manufacturing sector are higher than those in service sector, the production and labor in service tends to rise over time. As Lee and Wolpin (2006) [16], Chanda and Dalgaard (2005) [7], Buera and Koboski (2009) [5], and others point out, one of the most striking features over last fifty years in advanced economies is the rapid growth of service sector. This is consistent with the findings by Kuznets (1966) [13] and Chenery (1960) [8] that the sectoral composition changes over time but in the same way across most economies as they develop. The Kuznets fact suggests that economic growth requires structural change from agriculture to manufacturing and then to services. Recently, literatures studies undertake in integrating the Kuznets facts and the Kaldor (1963) [12]'s stylized fact. There are some literatures about structural change and economic growth. On the one hand, literatures such as Baumol (1967) [3] and Acemoglue and Guerrieri (2008) [1] focus on technology-related (supply side) reason to explain structural change and economic growth. On the other hand, literatures such as Kongsamut, Rebelo, and Xie (2001) [14], Laitner (2000) [15], Gollin, Parente, and Rogerson (2000) [9], Goodfriend and McDermott (1995) [10], and Alvarez-Cuadrado and Poschke (2011) [2] focus on preference-related (demand side) reason explain structural change and economic growth. Ngai and Pissarides (2007) [18] provides an explanation of structural change and economic growth both on supply side and on demand side. Caselli and Coleman (2001) [6] studies regional convergence and structural transformation from agriculture to manufacturing in a two-sector dynamic stochastic general equilibrium model.

Brida and Accinelli (2007) [4] constructs the one-sector optimal growth model with logistic population growth, and they show that logistic population growth does not affect the steady state. See also Guerrini (2010) [11] and Ferrara and Gierroni (2009) [17] as for the logistic population growth and one-sector optimal growth model. In our paper, we construct a two-sector optimal growth model consisting of service and manufacturing. You may rein-terpret consumption goods as service and investment goods as manufacturing goods in the model of Uzawa (1964) [19]. But in our model, services are only consumed while manufacturing goods are used for consumption and investment. We assume that the technology in services and manufacturing sector has different sigmoid process. Unlike previous literatures, without assuming the non-homothetic utility function, we verify "Baumol's cost disease" and the Kuznets fact. There is no literature studying the sector-specific logistic technological progress in the two-sector model. Our two-sector model also assume the logistic population growth. We obtain three main results as follows. First, the logistic population growth does not affect the steady state. On the other hand, the logistic sector-specific technology of service and of manufacturing goods industry has an asymmetric effect on the steady state. Second, the carry capacity of the service sector-specific technology has a negative effect on the relative price of service at the steady state while it does not affect the capital per capita at the steady state. On the other hand, the carry capacity of the manufacturing sector-specific technology has a positive effect on the relative price of service and capital stock per capita at the steady state. According to the Stolper-Samuelson theorem, the increase in the relative service price increases the wage rate in the case that the service sector is labor-intensive. Therefore the service industry absorbs more labor eventually. Finally, the dynamical system in the two sector optimal growth model with logistic sector-specific technological progress and population growth has saddle point stability.

2 The Model

2.1 Households: The expenditure Function

Let us consider a household which consumes two goods. Denote the consumption of goods 1 and goods 2 by c_t and x_t, respectively. Suppose that goods 1 is service, and goods 2 is manufacturing goods. The utility function of an agent is assumed to be $u_t = u(c_t, x_t) = c_t^{\eta} x_t^{1-\eta}$ where $\eta \in (0, 1)$ is a preference parameter wighted to service. We assume that goods 2 is numeral. The relative price of service in terms of manufacturing goods is P_t. The expenditure minimization problem of a household is given by $\min_{c,x} P_t c_t + x_t$, subject to $u_t = c_t^{\eta} x_t^{1-\eta}$. Then, the demand for goods 1 and goods 2 are derived as follows: $c_t = \eta^{1-\eta}(1-\eta)^{\eta-1} P_t^{\eta-1} u_t$ and $x_t = \eta^{-\eta}(1-\eta)^{\eta} P_t^{\eta} u_t$. The expenditure function is given by $E(P_t, u_t) = \eta^{-\eta}(1-\eta)^{\eta-1} P_t^{\eta} u_t$. According to the McKenzie's lemma (the envelop theorem), the demand for goods 1 is given by $\frac{\partial E(P_t, u_t)}{\partial P_t}$, and the demand for goods 2 is given by $E(P_t, u_t) - P_t \frac{\partial E(P_t, u_t)}{\partial P_t}$.

2.2 Firms: The GDP function

There are two industries where perfect competitive firms produce goods. Industry 1 produces services. Firms in the industry 1 demand capital K_{1t} and labor L_{1t} and produce services Y_{1t} by using the Cobb-Douglas production technologies $Y_{1t} = A_t K_{1t}^{\alpha} L_{1t}^{1-\alpha}$, where A_t is sector-specific technology in the service industry. Industry 2 produces manufacturing goods. Firms in the industry 2 demand capital K_{2t} and labor L_{2t} and produces manufacturing goods Y_{2t} by using the production technologies $Y_{2t} = Z_t K_{2t}^{\beta} L_{2t}^{1-\beta}$, where Z_t is sector-specific technology in the manufacturing industry. Denote a rental rate of capital by R_t and a wage rate by w_t. Then we can obtain the unit cost function $\phi^1(R_t, w_t)$ of firms in the industry 1 which is the cost required to produce one service, and the unit cost function $\phi^2(R_t, w_t)$ of firms in the industry 2. The unit cost function of each industry is given respectively by $\phi^1(R_t, w_t) \equiv$

$R_t \dfrac{K_{1t}}{Y_{1t}} + w_t \dfrac{L_{1t}}{Y_{1t}} = A_t^{-1}\Upsilon_1 R_t^{\alpha} w_t^{1-\alpha}$ and $\phi^2(R_t, w_t) \equiv R_t \dfrac{K_{2t}}{Y_{2t}} + w_t \dfrac{L_{2t}}{Y_{2t}} = Z_t^{-1}\Upsilon_2 R_t^{\beta} w_t^{1-\beta}$, where the constant is given respectively by $\Upsilon_1 \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$ and $\Upsilon_2 \equiv \beta^{-\beta}(1-\beta)^{\beta-1}$. The factor market equilibrium of capital and labor is given respectively by $K_{1t} + K_{2t} = K_t$ and $L_{1t} + L_{2t} = L_t$ where K_t is total capital and L_t is total labor. We assume that the manufacturing good is numeral. Denote the relative price of service in terms of manufacturing goods by P_t. The profit maximization condition gives $P_t = \phi^1(R_t, w_t)$ and $1 = \phi^2(R_t, w_t)$.

The duality theory gives a definition of the GDP function as follows;

$$Y(P_t, K_t, L_t) = \min_{R_t, w_t} R_t K_t + w_t L_t, \quad \text{s.t.} \quad P_t = A_t^{-1}\Upsilon_1 R_t^{\alpha} w_t^{1-\alpha}, \quad 1 = Z_t^{-1}\Upsilon_2 R_t^{\beta} w_t^{1-\beta}.$$

We can show that GDP function is given by

$$Y(P_t, K_t, L_t) = R_t(P_t)K_t + w_t(P_t)L_t = \Omega_K A_t^{\frac{\beta-1}{\beta-\alpha}} Z_t^{\frac{1-\alpha}{\beta-\alpha}} P_t^{\frac{\beta-1}{\beta-\alpha}} K_t + \Omega_L A_t^{\frac{\beta}{\beta-\alpha}} Z_t^{\frac{-\alpha}{\beta-\alpha}} P^{\frac{\beta}{\beta-\alpha}} L_t \tag{1}$$

where constants are given by $\Omega_K \equiv \Upsilon_1^{\frac{1-\beta}{\beta-\alpha}} \Upsilon_2^{\frac{\alpha-1}{\beta-\alpha}}$ and $\Omega_L \equiv \Upsilon_1^{\frac{-\beta}{\beta-\alpha}} \Upsilon_2^{\frac{\alpha}{\beta-\alpha}}$. Note that GDP function is measured in terms of manufacturing goods. Denote the capital stock per capita by $k_t \equiv \frac{K_t}{L_t}$, and denote the per capita GDP by $y_t \equiv \frac{Y_t}{L_t}$. The per capita Cobb-Douglas GDP function is given by

$$y(P_t, k_t) = R_t(P_t)k_t + w_t(P_t) = \Omega_K A_t^{\frac{\beta-1}{\beta-\alpha}} Z_t^{\frac{1-\alpha}{\beta-\alpha}} P_t^{\frac{\beta-1}{\beta-\alpha}} k_t + \Omega_L A_t^{\frac{\beta}{\beta-\alpha}} Z_t^{\frac{-\alpha}{\beta-\alpha}} P_t^{\frac{\beta}{\beta-\alpha}}. \tag{2}$$

The rental rate of capital R_t and the wage rate w_t in terms of manufacturing goods is given respectively by $R_t(P_t) = \Omega_K A_t^{\frac{\beta-1}{\beta-\alpha}} Z_t^{\frac{1-\alpha}{\beta-\alpha}} P_t^{\frac{\beta-1}{\beta-\alpha}}$ and $w_t(P_t) = \Omega_L A_t^{\frac{\beta}{\beta-\alpha}} Z_t^{\frac{-\alpha}{\beta-\alpha}} P_t^{\frac{\beta}{\beta-\alpha}}$.

2.3 Sector-Specific Logistic Technological Progress and Population Growth

We assume that the sector-specific technology is depicted by a logistic growth law. The service sector-specific technology A_t follows $\dfrac{\dot{A}_t}{A_t} = \lambda\left(1 - \dfrac{A_t}{A_\infty}\right)$, where λ is a exponential growth rate and A_∞ is the carrying capacity or the saturation of sigmoidal process. At the same way, the manufacturing goods sector-specific technology Z_t follows $\dfrac{\dot{Z}_t}{Z_t} = \psi\left(1 - \dfrac{Z_t}{Z_\infty}\right)$, where ψ is a exponential growth rate of technology in the manufacturing sector and Z_∞ is the carrying capacity or the saturation. At the steady state, both rates of sector-specific technical progress become zero.

As population becomes so large, food shortage or crowding occurs. Therefore, population growth is bounded. Population L_t follows $n(t) = \dfrac{\dot{L}_t}{L_t} = \gamma\left(1 - \dfrac{L_t}{L_\infty}\right)$, where γ is the Malthusian coefficient and L_∞ is the carrying capacity or the limit for the population size. $n(t)$ is the variable rate of population growth.

2.4 Two-Sector Optimal Growth Model

We consider a representative agent model with infinite horizon. The dynamic optimization problem is given by $\int_0^\infty e^{-\rho t}u_t dt$, subject to $\dot{k}_t = y(P_t, k_t) - E(P_t, u_t) - \frac{\dot{L}_t}{L_t}k_t$, and subject to $\dot{A}_t = \lambda A_t\left(1 - \frac{A_t}{A_\infty}\right), \dot{Z}_t = \psi Z_t\left(1 - \frac{Z_t}{Z_\infty}\right)$, and $\dot{L}_t = \gamma L_t\left(1 - \frac{L_t}{L_\infty}\right)$, where $\rho > 0$ is the discount rate. Recall the expenditure function $E(P_t, u_t) = \eta^{-\eta}(1-\eta)^{\eta-1}P_t^\eta u_t$, and the per capita GDP function $y(P_t, k_t) = R_t(P_t)k_t + w_t(P_t) = \Omega_K A_t^{\frac{\beta-1}{\beta-\alpha}} Z_t^{\frac{1-\alpha}{\beta-\alpha}} P_t^{\frac{\beta-1}{\beta-\alpha}} k_t + \Omega_L A_t^{\frac{\beta}{\beta-\alpha}} Z_t^{\frac{-\alpha}{\beta-\alpha}} P_t^{\frac{\beta}{\beta-\alpha}}$.

2.5 The optimality condition and market equilibrium

We obtain the intertemporal optimality condition and the per capita capital accumulation as follows: $\frac{\dot{P}_t}{P_t} = \frac{1}{\eta}\left[R(P_t) - \rho - \gamma\left(1 - \frac{L_t}{L_\infty}\right)\right]$, and $\dot{k} = y(P_t, k_t) - E(P_t, u_t) - \gamma\left(1 - \frac{L_t}{L_\infty}\right)k_t$.

The market equilibrium condition of services is given by $\frac{\partial y(P_t,k_t)}{\partial P_t} = \frac{\partial E(P_t,u_t)}{\partial P_t}$, and the market equilibrium condition of manufacturing goods is expressed as $y(P_t, k_t) - P_t\frac{\partial y(P_t,k_t)}{\partial P_t} = E(P_t, u_t) - P_t\frac{\partial E(P_t,u_t)}{\partial P_t}$. From the market equilibrium condition of services, we have $P_t\frac{\partial y(P_t,k_t)}{\partial P_t} = P_t\frac{\partial E(P_t,u_t)}{\partial P_t}$, or rewritten as follows:

$$P_t\frac{\partial y(P_t,k_t)}{\partial P_t} = \Omega_K A_t^{\frac{\beta-1}{\beta-\alpha}} Z_t^{\frac{1-\alpha}{\beta-\alpha}}\left(\frac{\beta-1}{\beta-\alpha}\right)P_t^{\frac{\beta-1}{\beta-\alpha}}k_t + \Omega_L A_t^{\frac{\beta}{\beta-\alpha}} Z_t^{\frac{-\alpha}{\beta-\alpha}}\left(\frac{\beta}{\beta-\alpha}\right)P_t^{\frac{\beta}{\beta-\alpha}}$$

$$= \eta\eta^{-\eta}(1-\eta)^{\eta-1}P_t^\eta u_t = P_t\frac{\partial E(P_t,u_t)}{\partial P_t} = \eta E(P_t,u_t).$$

Therefore we obtain $E(P_t, u_t) = \frac{1}{\eta}P_t\frac{\partial y(P_t,k_t)}{\partial P_t}$, or rewritten as follows:

$$E(P_t,u_t) = \frac{1}{\eta}\left(\Omega_K A_t^{\frac{\beta-1}{\beta-\alpha}} Z_t^{\frac{1-\alpha}{\beta-\alpha}}\left(\frac{\beta-1}{\beta-\alpha}\right)P_t^{\frac{\beta-1}{\beta-\alpha}}k_t + \Omega_L A_t^{\frac{\beta}{\beta-\alpha}} Z_t^{\frac{-\alpha}{\beta-\alpha}}\left(\frac{\beta}{\beta-a}\right)P_t^{\frac{\beta}{\beta-\alpha}}\right). \quad (3)$$

Thus we obtain the capital accumulation equation as follows: $\dot{k} = y(P_t, k_t) - \frac{P_t}{\eta}\frac{\partial y(P_t,k_t)}{\partial P_t} -$

$\gamma\left(1-\frac{L_t}{L_\infty}\right)k_t$.

2.6 The dynamical system and steady state

We describe the dynamical system in our two-sector optimal growth model as follows;

$$\dot{P}_t=\frac{P_t}{\eta}\left[\Omega_K A_t^{\frac{\beta-1}{\beta-\alpha}} Z_t^{\frac{1-\alpha}{\beta-\alpha}} P_t^{\frac{\beta-1}{\beta-\alpha}}-\rho-\gamma\left(1-\frac{L_t}{L_\infty}\right)\right],\ \dot{k}=\Omega_K A_t^{\frac{\beta-1}{\beta-\alpha}} Z_t^{\frac{1-\alpha}{\beta-\alpha}}\left(\frac{1-\alpha\eta-(1-\eta)\beta}{\eta(\beta-\alpha)}\right)P_t^{\frac{\beta-1}{\beta-\alpha}}k_t+$$

$$\Omega_L A_t^{\frac{\beta}{\beta-\alpha}} Z_t^{\frac{-\alpha}{\beta-\alpha}}\left(\frac{-\alpha\eta-(1-\eta)\beta}{\eta(\beta-\alpha)}\right)P^{\frac{\beta}{\beta-\alpha}}-\gamma\left(1-\frac{L_t}{L_\infty}\right)k_t,\ \dot{A}_t=\lambda A_t\left(1-\frac{A_t}{A_\infty}\right),\ \dot{Z}_t=\psi Z_t\left(1-\frac{Z_t}{Z_\infty}\right),$$

and $\dot{L}_t=\gamma L_t\left(1-\frac{L_t}{L_\infty}\right)$, where the initial capital stock per capita k_0, the initial technological level of service A_0 and of manufacturing industry Z_0, the initial population L_0, the rate of growth rate λ, ψ, and γ, the carrying capacity A_∞, Z_∞, and L_∞ are given exogenously, and note $1-\alpha\eta-(1-\eta)\beta>0$ and $-\alpha\eta-(1-\eta)\beta<0$. As Ferrara, M. and Gierroni, L. (2009) [17] points out, if we supposed the Benthamite felicity function, the Euler equation would not include the rate of population growth in our model.

The $\dot{P}=0$ locus is $P=\left(\frac{\Omega_K}{\rho}\right)^{\frac{\beta-\alpha}{1-\beta}}A^{-1}Z^{\frac{1-\alpha}{1-\beta}}$ and the $\dot{k}=0$ locus is given by $P=\left(\frac{Z}{A}\right)\left(\frac{\alpha\eta+(1-\eta)\beta}{1-\alpha\eta-(1-\eta)\beta}\right)^{\alpha-\beta}\left(\frac{\Omega_K}{\Omega_L}\right)^{\beta-\alpha}k^{\beta-\alpha}$. At the steady state, $R(P^*)=\rho$, $y(P^*,k^*)=\frac{P^*}{\eta}\frac{\partial y(P^*,k^*)}{\partial P}$, $A^*=A_\infty$, $Z^*=N$, and $L^*=L_\infty$ hold. We obtain the relative price of service and capital per capita at the steady state as follows: $P^*=\left(\frac{\Omega K}{\rho}\right)^{\frac{\beta-\alpha}{1-\beta}}A_\infty^{-1}Z_\infty^{\frac{1-\alpha}{1-\beta}}$, and $k^*=\left(\frac{Z\infty}{\rho}\right)^{\frac{1}{1-\beta}}\Omega_K^{\frac{\beta}{1-\beta}}\Omega_L\left(\frac{\alpha\eta+(1-\eta)\beta}{1-\alpha\eta-(1-\eta)\beta}\right)$, where note that the sector-specific logistic technology eventually reaches the carrying capacity $A^*=A_\infty$ and $Z^*=Z_\infty$. The effect of the carrying capacity of the sector-specific technology on the relative price at the steady state is given by $\frac{\partial P^*}{\partial A_\infty}<0$ and $\frac{\partial P^*}{\partial Z_\infty}>0$. The effect of the carrying capacity of the sector-specific technology on the capital per capita at the steady state is given by and $\frac{\partial k^*}{\partial A_\infty}=0$ and $\frac{\partial k^*}{\partial Z_\infty}>0$. The relative service price and capital stock per capita at the steady state does not depend on population. Therefore the logistic population growth does not affect the steady state even in the two-sector optimal growth model.

Note that an increase in the relative service price increases the supply for services because of $\frac{\partial y(P,k)}{\partial P}>0$ and $\frac{\partial^2 y(P,k)}{\partial P^2}>0$. The effect of the sector-specific logistic technology on the relative service price and on the supply for services is shown as follows. The result of $\frac{\partial P^*}{\partial A_\infty} < 0$ suggests that the increases in the carry capacity of the service sector-specific technology decreases the relative price and the supply for services. On the other hand, the result of $\frac{\partial P^*}{\partial Z_\infty} > 0$ suggests that the increases in the carry capacity of the manufacturing sector-specific technology increases the relative price and the supply for services. This result is expressed as Baumol's cost disease. As Baumol (1967) emphasizes, technology of the manufacturing sector gives rise to the movement of labor in the direction of the service sector. According to the Stolper-Samuelson theorem, the relative service price increases the wage rate in the case that the service sector is labor-intensive. Therefore the service industry absorbs more labor eventually.

The effect of the sector-specific logistic technology on the capital stock per capita at the steady state is examined as follows. $\frac{\partial k^*}{\partial A_\infty} = 0$ implies that the upper limit of the service sector-specific technology does not affect capital stock per capita at the steady state. On the other hand, $\frac{\partial k^*}{\partial Z_\infty} > 0$ suggests that the increases in the carry capacity of the manufacturing sector-specific technology increases capital stock per capita at the steady state. These results explained above as for the Baumol's cost disease and the structural change from manufacturing to service are summarized as follows.

Proposition 1

The logistic population growth does not affect the steady state. On the other hand, the logistic sector-specific technology of service and of manufacturing goods industry has an asymmetric effect on the steady state.

Proposition 2

The carry capacity of the service sector-specific technology has a negative effect on the relative price of service at the steady state while it does not affect the capital per capita at the steady state. On the other hand, the carry capacity of the manufacturing sector-specific technology has a positive effect on the relative price of service and capital stock per capita at the steady state.

The proposition 2 is the most important in our paper and it complements the Baumal's insistence.

2.7 The dynamic stability

By carrying out the linerization of the dynamical system around the steady state, we obtain

$$\begin{pmatrix} \dot{P} \\ \dot{k} \\ \dot{A} \\ \dot{Z} \\ \dot{L} \end{pmatrix} = \begin{pmatrix} J_{11}, & 0, & \frac{P^*}{\eta}\frac{\partial R(P^*)}{\partial A_t}, & \frac{P^*}{\eta}\frac{\partial R(P^*)}{\partial Z_t}, & \frac{P^*}{\eta}\frac{\partial R(P^*)}{\partial L_t} \\ J_{21} & J_{22} & J_{23} & J_{24} & \frac{\gamma}{L_\infty}k^* \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -\psi & 0 \\ 0 & 0 & 0 & 0 & -\gamma \end{pmatrix} \begin{pmatrix} P_t - P^* \\ k_t - k^* \\ A_t - A^* \\ Z_t - Z^* \\ L_t - L^* \end{pmatrix}, \tag{4}$$

where

$$J_{11} = \frac{P^*}{\eta}\frac{\partial R(P^*)}{\partial P_t} \begin{cases} >0, \text{ if } \alpha>\beta \\ <0, \text{ if } \alpha<\beta \end{cases} \tag{5}$$

$$J_{21} = \frac{1}{\eta}\left[-(1-\eta)\frac{\partial y(P^*,k^*)}{\partial P_t} - P^*\frac{\partial^2 y(P^*,k^*)}{\partial P_t^2} \right] < 0, \tag{6}$$

$$J_{22} = \frac{\partial R(P^*)}{\partial P_t} - \frac{P^*}{\eta}\frac{\partial R(P^*)}{\partial P_t} \begin{cases} <0, \text{ if } \alpha>\beta \\ >0, \text{ if } \alpha<\beta \end{cases}, \tag{7}$$

$$J_{23} = \frac{\partial y(P^*,k^*)}{\partial A_t} - \frac{P^*}{\eta}\frac{\partial^2 y(P^*,k^*)}{\partial P_t \partial A_t}, \tag{8}$$

$$J_{24} = \frac{\partial y(P^*,k^*)}{\partial Z_t} - \frac{P^*}{\eta}\frac{\partial^2 y(P^*,k^*)}{\partial P_t \partial Z_t}. \tag{9}$$

Denote the Jacobian in the dynamical system by J, and denote the eigenvalues of J by μ. The characteristic equation is expressed as

$$\Psi_J(\mu) = (\mu - J_{11})(\mu - J_{22})(\mu - (-\lambda))(\mu - (-\psi))(\mu - (-\gamma)). \tag{10}$$

There are five eigenvalues. If μ_1 is positive, then μ_2 is negative, and vice versa. This is because J_{11} is positive and J_{22} is negative in the case that the service is capital-intensive, while J_{11} is negative and J_{22} is positive in the case that the service is labor-intensive. The number of state variables (k_t, A_t, Z_t, L_t) is four and the number of negative eigenvalues is four. Therefore the dynamic system has saddle point stability.

Proposition 3

The dynamical system in the two sector optimal growth model with logistic sector-specific technological progress and population growth has saddle point stability.

3 Conclusion

We have developed a two-sector optimal growth model not only with logistic population growth but also with sector-specific logistic technological progress. Without assuming the non-homothetic utility function, we have verified the structural change from manufacturing to services, as pointed by Baumol and Kzunets. Our paper provides an explanation of structural change and economic growth on supply side. We have shown that the logistic population growth does not affect the steady state while the logistic sector-specific technology of service and of manufacturing goods industry has an asymmetric effect on the steady state. Moreover, we show that the carry capacity of the service sector-specific technology has a negative effect on the relative price of service at the steady state while it does not affect the capital per capita at the steady state. In other words, an increase in the upper limit of technology in the service sector decreases the relative service price and the supply for services, and it does not facilitate capital accumulation. On the other hand, the carry capacity of the manufacturing sector-specific technology has a positive effect on the relative price of service and capital stock per capita at the steady state. As Baumol emphasizes, technology of the manufacturing sector gives rise to the movement of labor in the direction of the service sector. According to the Stolper-Samuelson theorem, the relative service price increases the wage rate in the case that the service sector is labor-intensive. Therefore the service industry absorbs more labor eventually. We have explained the Baumol's cost disease and the structural change from manufacturing to services.

* This work is supported by a Grant-in-Aid for Young Scientists (WAKATE B - 16K51525) from the Ministry of Education, Culture, Sports, Science and Technology, Japan.
† Faculty of Department of Economics, Meiji Gakuin University.
‡ 1-2-37 Shirokanedai, Minato-ku, 108-8636, Tokyo, Japan.
§ E-mail: muro@eco.meijigakuin.ac.jp

References

[1] Acemoglue, D. and Guerrieri, V. (2008), "Capital Deepening and Nonbalanced Economic Growth", *Journal of Political Economy* 116(3), 467–498.

[2] Alvarez-Cuadrado, F. and Poschke, M. (2011), "Structural Change Out of Agriculture: Labor Push versus Labor Pull", *American Economic Journal: Macroeconomics* 3, 127–158.

[3] Baumol, W.J. (1967), "Macroeconomics of unbalanced growth: the anatomy of urban crisis", *The American Economic Review*.

[4] Brida, J.G., and Accinelli, E. (2007), "The Ramsey Model with Logistic Population Growth", *Economic Bullitain* 3(15), 1–8.

[5] Buera and Koboski. (2009), "The Rise of the Service Economy", *NBER Working paper* 14822.

[6] Caselli, F. and Coleman, W.J. (2001), "The U.S. Structural Transformation and Regional Convergence: A Rein-

terpretation", *Journal of Political Economy* 109(3), 584–616.

[7] Chanda, A. and Dalgaard, C.J. (2005) "Wage Inequality and the Rise of Services," *mimeo*, Louisiana State University.

[8] Chenery, H. (1960), "Patterns of Industrial Growth", *American Economic Review* 50, 624–654.

[9] Gollin, D., Parente, S., and Rogerson, R. (2000), "The role of Agriculture in Development", *American Economic Review papers and Proceedings* 92(2), 160–164.

[10] Goodfriend, M. and McDermott, J. (1995), "Early Development", *American Economic Review* 85, 116–133.

[11] Guerrini, L. (2010), "A closed-form solution to the Ramsey model with logistic population growth", *Economic Modeling* 27(5), 1178–1182.

[12] Kaldor, N. (1963), "Capital accumulation and economic growth", in Friedrich A. Lutz and Douglas C. Hague, eds., Proceedings of a Conference Held by the Inter-national Economics Association, London, *MacMillan*.

[13] Kuznets, S. (1966), "Modern Economic Growth", *Yale University Press*.

[14] Kongsamut, P., Rebelo, R., and Xie, D. (2001), "Beyond balanced Growth", *The Review of Economic Studies* 68(4), 869-882.

[15] Laitner, J. (2000), "Structural Change and Economic Growth", *The Review of Economic Studies* 67, 545–561.

[16] Lee, D. and Wolpin, K. (2006), "Intersectoral labor Mobility and the Growth of the Service Sector", *Econometirca* 74(1), 1–46.

[17] Ferrara, M. and Gierroni, L. (2009), "The Ramsey Model with Logistic Population Growth and Benthamite Felicity Function", *Public Choice and Political Economy*.

[18] Ngai, L.R. and Pissarides, C.A. (2007), "Structural change in a multisector model of growth", *The American Economic Review* 97(1), 429–443.

[19] Uzawa, H. (1964), "Optimal growth in a two-sector model of capital accumulation", *Review of Economic Studies*.